

Examples of non-homomorphisms

| Non-homomorphism | Cases which 'work' | $\phi(\text{identity}) = \text{identity?}$ |
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| 1. $\phi: (\mathbb{C}^*, \times) \rightarrow (\mathbb{C}, +)$ $z \mapsto z + \operatorname{Re}(z)$ | $\phi(z_1 z_2) = \phi(z_1) + \phi(z_2)$ if $z_1 = z_2 = 2$ | No |
| 2. $\phi: (\mathbb{C}^*, \times) \rightarrow (\mathbb{C}^*, \times)$ $z \mapsto z \operatorname{Re}(z)$ | $\phi(z_1 z_2) = \phi(z_1) \phi(z_2)$ if $z_1 = z_2 = 1$ | Yes |
| 3. $\phi: (\mathbb{C}^*, \times) \rightarrow (\mathbb{C}^*, \times)$ $z \mapsto e^{iz}$ | $\phi(z_1 z_2) = \phi(z_1) \phi(z_2)$ if $z_1 = z_2 = 2$ | No |
| 4. $\phi: (\mathbb{R}, +) \rightarrow (\mathbb{R}, +)$ $x \mapsto \sin(x)$ | $\phi(x_1 + x_2) = \phi(x_1) + \phi(x_2)$ if $x_2 = n\pi$ | Yes |
| 5. $\phi: (\mathbb{R}, +) \rightarrow (\mathbb{R}, +)$ $x \mapsto 2^x$ | $\phi(x_1 + x_2) = \phi(x_1) + \phi(x_2)$ if $x_1 = x_2 = 1$ | No |
| 6. $\phi: (\mathbb{R}^*, \times) \rightarrow (\mathbb{R}^*, \times)$ $x \mapsto 2^x$ | $\phi(x_1 x_2) = \phi(x_1) \phi(x_2)$ if $x_1 = x_2 = 2$ | No |
| 7. $\phi: (\mathbb{R}^*, \times) \rightarrow (\mathbb{R}, +)$ $x \mapsto x$ | $\phi(xy) = \phi(x) + \phi(y)$ if $y = \frac{x}{x-1}$ eg $x = y = 2$ | No |
| 8. G a group of matrices under multiplication $\phi: G \rightarrow G$ $A \mapsto A^2$ | $\phi(AB) = \phi(A)\phi(B)$ if A, B commute | Yes |
| 9. G a group of matrices under addition $\phi: G \rightarrow (\mathbb{R}, +)$ $A \mapsto \det(A)$ | $\phi(A + B) = \phi(A) + \phi(B)$ if $A = B = 0$ or $B = -A$ has an odd number of rows/columns | Yes |
| 10. $\phi: S(\Delta) \rightarrow S(\Delta)$ $g \mapsto g^{-1}$ | $\phi(g_1 \circ g_2) = \phi(g_1) \circ \phi(g_2)$ if g_1, g_2 commute | Yes |